Problem of the Month
Pick a Pocket

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *Pick a Pocket,* students use mathematical concepts of statistics including average, consistency, reliability, and optimization. The mathematical topics that underlie this POM are measures of center, measures of dispersion, the distribution of data, and the interpretation of data sets.

In the first levels of the POM, students collect data and examine data sets around range and measures of center. As one continues through the levels, students

analyze problems to determine a mathematical definition of typical for a particular situation. In the final levels of the POM, students examine data in situations that require statistical tools other than measures of center. In the context of these situations students focus on measures of dispersion and the distribution of data.

Problem of the Month

Pick a Pocket

Level A:

Examine the number of pockets each student in your class is wearing.

Count the number of pockets worn by each student.

Make a table of the number of pockets worn by each student.

Draw a graph using the data from the table.

What is the largest amount of pockets worn by a student?

What is the smallest number of pockets anyone wore?

What is the total number of pockets worn by all the students?

Tomorrow, if we surveyed the class would we have the same number of pockets? Explain.

Level B:

What is the mode (most common number of pockets worn by the students)?

Find the mean average of pockets in your class.

Find the median number of pockets in your class.

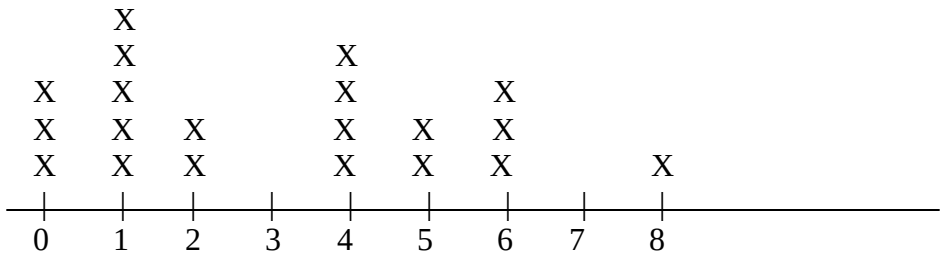
Find the range of the data (the largest number minus the smallest number).

If a new student walked into our class, how many pockets might the new student be wearing? Which mathematical measure might be the best one to use for such a prediction? Explain your answer.

Create a new set of data, different from your class, that has the same mean and median as your class data.

Level C:

In Dia's class they made a line plot of the different number of pockets worn by the students.



Each student wrote what they thought was the typical number of pockets for a student in their class.

Explain why or why not you think each statement make sense mathematically.

Raul: I think 3.7 is typical because I found the average by adding the numbers that have X's on them like 0+1+2+4+5+6+8 = 26 and 26 divided by the 7 numbers = 3.7

Joanne: I found the median but first I ignored each zero, since they don't have pockets. Therefore, there are 17 other X's and the ninth one would be in the middle. I started counting X's from the right side moving left and I landed on the X over the number 4. So, I think 4 pockets are typical for our class.

Austin: I think 8 pockets are way out of the normal range, so I am going to ignore it. Therefore, the numbers that have pockets go from 0 to 6 and (0 + 6)/2 = 3. Therefore, 3 is my choice for most typical, because it is in the middle.

Ming: I calculated the following way to find an answer.
3 x 0 + 5 x 1 + 2 x 2 + 4 x 4 + 2 x 5 + 3 x 6 + 1 x 8 = 61
61 / 20 = 3.1 I think 3 is typical.

Level D:

You just got a new video game called Drop in the Pocket. It simulates a new type of game of pool. There is one pocket (hole) in the middle of the table. You hit a ball from a starting point trying to sink it in the pocket. If you miss it, the game tells you how many millimeters you are from the pocket. You and your two friends each try playing by taking turns for five rounds. None of you actually make it in the pocket. At the end of the round the game shows the results.

Round	Player A	Player B	Player C
1	100	184	99
2	40	64	200
3	312	76	165
4	60	52	129
5	152	288	84

Use mathematics to determine who was the best player over the five rounds. Explain a method or system that might be used to judge a winner or keep score of the game.

Level E:

You are a manager for a clothing manufacturer. You work in the department that makes denim pants. You have workers who sew pockets onto the pants. You want to award one of them as Employee of the Month. So for twelve days you track the number of pants they sew each day. Below are the numbers of completed pants by the two most productive employees for each of the 12 days. Use mathematics to determine which of the two employees should get the award. Remember you might have to explain to the other employee why they didn't get chosen so you must use a mathematical measure to be most objective.

Day	Employee Ariana	Employee Brian
1	32	49
2	75	45
3	38	51
4	42	49
5	47	63
6	68	56
7	51	51
8	51	48
9	58	52
10	31	42
11	51	51
12	65	52

Problem of the Month

Pick a Pocket

Primary Version Level A

Materials: A large poster with the scale for a line, a Post-it note for each student.

Discussion on the rug: (Teacher points to a pocket that someone is wearing) "What do you call this? "What is it used for?" (Teacher solicits answers from students) "How many pockets do you think are in this class?" Why did you make that guess?" (Teacher solicits answers from students) "How might we find out how many pockets each of us have and all of us have together?" (Teacher solicits answers from students and then states that the class will investigate this question)

In small groups: (Each student gets sticky note. They write the number of pockets they have on the note. Students one by one come up to a line plot and place their note over the corresponding number.) "Looking at our graph who can tell me… Which student(s) have the most pockets?
Which students have the least number of pockets?
How many pockets do we have in total?"
"If we had a new student come into our class, how many pockets do you think that student might wear? Explain why you think it would be that number."
(At the end of the investigation have students either discuss or dictate a response to this summary question)

Problem of the Month

Pick a Pocket

Task Description – Level A

This task challenges a student to collect data about the number of pockets each student has and put the data in a table. Students are then challenged to draw a graph with the data and interpret data from the graph, such as least, most, and total number of pockets. Students are also questioned about the reliability of the data for predicting future events.

Common Core State Standards Math - Content Standards

Measurement and Data

Represent and interpret data.

2.MD.10 Draw a picture graph and a bar graph (with a single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Pick a Pocket

Task Description – Level B

This task challenges a student to calculate measures of center from their bar graph and find the range of the data. Students then decide which measure best fits the data to make a prediction. Students then try to design a new set of data to give the same mean and median as the class set.

Common Core State Standards Math - Content Standards

Statistics and Probability

Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Use random sampling to draw inferences about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

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reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Pick a Pocket

Task Description – Level C

This task challenges a student to think about statistical measures in the context of a line plot. Students are confronted with some misconceptions and are asked to explain why they do or don't make sense.

Common Core State Standards Math - Content Standards

Measurement and Data

Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements.

Statistics and Probability

Develop understanding of statistical variability.

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6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

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Problem of the Month

Pick a Pocket

Task Description – Level D

This task challenges a student to compare three sets of data about playing a video game. Students use statistical measures to analyze the data in the context of the game to determine the better player.

Common Core State Standards Math - Content Standards

Statistics and Probability

Develop understanding of statistical variability.

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6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

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Problem of the Month

Pick a Pocket

Task Description – Level E

This task challenges a student to compare and contrast two data sets about employ performance to pick the employee of the month. Students must develop a convincing argument for their choice, including being able to justify why the other employee was not chosen.

Common Core State Standards Math - Content Standards

Statistics and Probability

Develop understanding of statistical variability.

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Problem of the Month

Pick a Pocket

Task Description – Primary Level

This task challenges a student to count pockets and place cards on a line plot for data collected. Students then answer questions about the plot, most, least, total number of pockets.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Compare numbers.

K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.1

K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less that or equal to 20, e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns n all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Measurement and Data

Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent data with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in the scaled bar graphs.

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